Challenges and Experiences in Managing Large-Scale Proofs

T. Bourke[1] *M. Daum*[1,2] G. Klein[1,2] R. Kolanski[1,2]

[1]NICTA, Sydney, Australia

[2]The University of NSW, Sydney, Australia

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A Sense of Scale

Basic Claim

Scale changes everything—be it code or mathematical proof.

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Some examples of **mechanised** proofs:

- four-colour theorem: around **60,000** lines in Coq
- higher-order logic (HOL) library in Isabelle: around **66,000** lines
- Archive of Formal Proofs (AFP):
  entries range between **145** and **80,917** lines in Isabelle
- CompCert verified compiler: about **100,000** lines in Coq

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- Archive of Formal Proofs (AFP):
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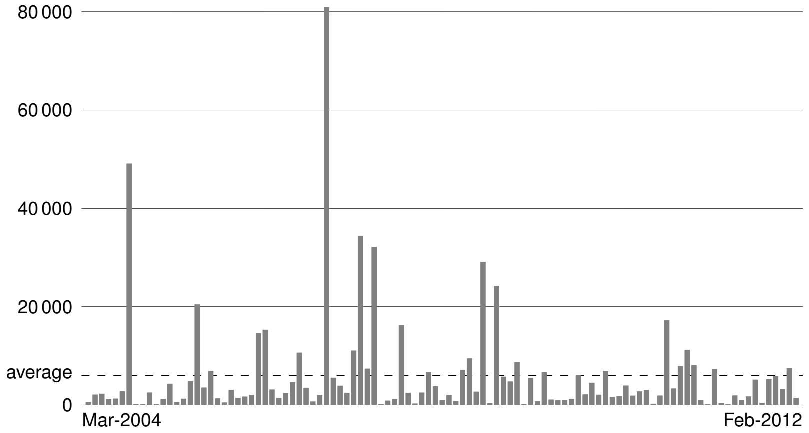
- CompCert verified compiler: about **100,000** lines in Coq
- L4.verified project repository: around **390,000** lines in Isabelle
- Verisoft project published over **500,000** lines in Isabelle

A Sense of Scale

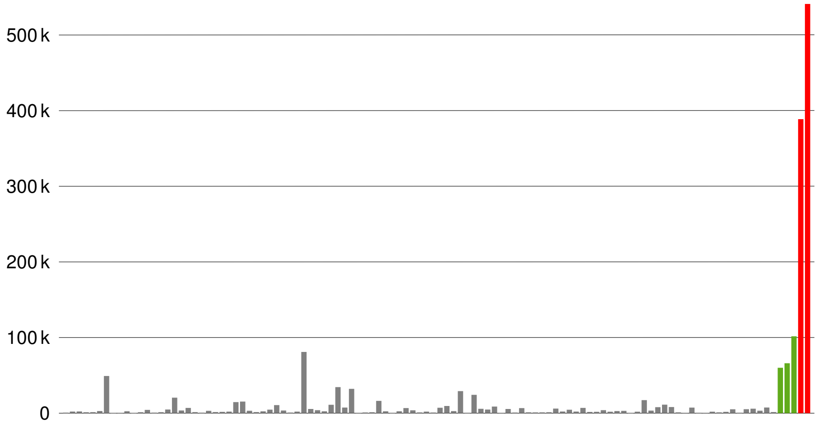
Size Distribution of AFP Entries



Size distribution of AFP entries in lines of proof, sorted by submission date

A Sense of Scale

Lines of Proof in Comparison



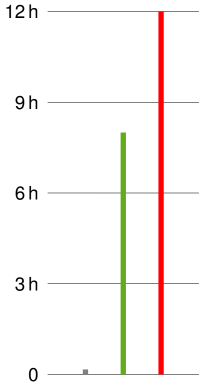
🟫 AFP entries by submission date
🟩 four-color theorem, Isabelle/HOL, CompCert
🟥 L4.verified, Verisoft

A Sense of Scale

Estimated Runtime for a Full Proof Check

- ⬛ Isabelle/HOL: 10 minutes
- 🟩 L4.verified: 8 hours
- 🟥 Verisoft: 12 hours

Note: checking times vary significantly with the utilization of the processor.



A Sense of Scale

Our Notion of “Large”

- different possible measures: lines, theorems, theories, . . .
- numbers vary with person, language, tool, problem, . . .
- hence: less stress on precise figures

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Classification

Large-scale developments

- concern multiple people over multiple years and
- no single person understands the whole proof at any given time.

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L4.verified: 12^+ people over 7^+ years

Overview: Challenges

Four perspectives:

- proof introspection – finding existing definitions and theorems
- proof development – proving new statements
- proof maintenance – keeping the proof base alive
- social and management aspects – from many brains to one proof

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Proof Introspection

Rafal's Observation

I still maintain that the introspection of proof and theories is an essential part of working on a large-scale verification development.

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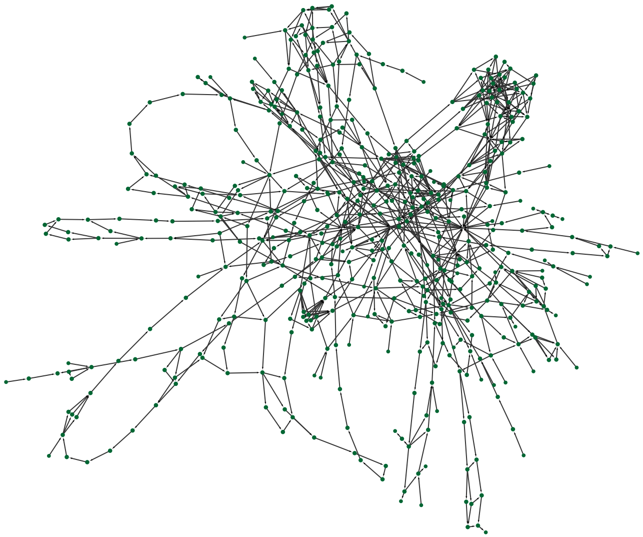
Our experience in training new team members:

- Learning Isabelle? – Easy.
- Understanding the verification subject? – No big deal.
- Understanding the proofs is the hard part!

Proof Introspection

How do you find your way through this jungle?



Theory-file dependencies in L4.verified

Proof Introspection

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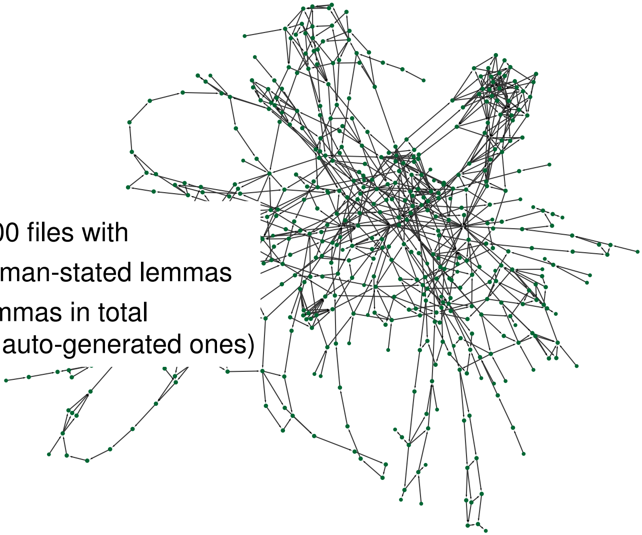

- roughly 500 files with
- 22,000 human-stated lemmas
- 95,000 lemmas in total (including auto-generated ones)

Theory-file dependencies in L4.verified

Proof Introspection

- `find_theorems` tool with
  - pattern-matching against theorem statements and names
  - filtering rules against current goal
  - ranking by most accurate match
- **auto-solve** function – warn if existing lemma is restated
- context-independent search over a **web-interface**
- **locate** tool[1] – find definitions; decode syntactic sugar

[1] inspired by Coq.

Proof Introspection

Solutions – L4.verified Tools

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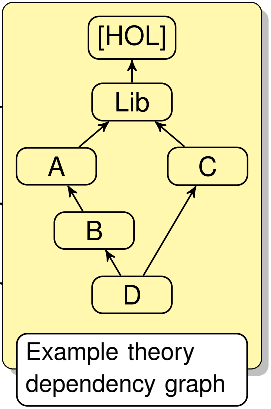

Example theory dependency graph

[1]inspired by Coq.

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```
lemma trancl_refcl: "(r⁼)⁺ = r*"
```

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Proof Development

Answer, Quick!

Progress

Proof checking in progress
Please wait...

7 h 56 min remaining

Proof Development

Matthias' Conjecture

Over the years, I must have waited weeks for Isabelle.
Productivity hinges on a short edit-check cycle;
for that, I am even willing to (temporarily) sacrifice soundness.

- Challenges:
  - non-local change
  - speculative change
  - distributed development
- Solutions:
  - skip-proof mode (Isabelle)
  - proof cache (L4.verified)
  - concurrency (recent improvement)

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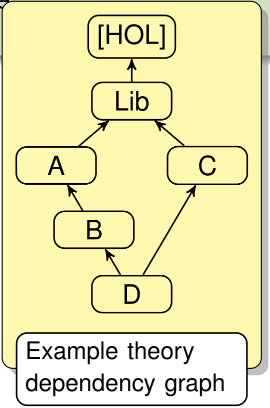
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Proof Development

`by auto`

Tim's Statement

Automating "donkey work" allows attention and effort to be focussed where most needed – but it must be done judiciously.

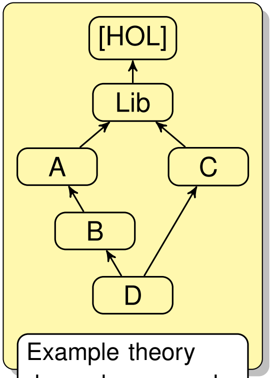
Focus: domain-specific automation

- extensions boost productivity
- unsoundness strikes back!

Proof Development

Common Problems

- lemma placement – solutions:
  - **Gravity** tool (Verisoft)
  - **Levity** tool (L4.verified)



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  - avoid – spot & mark – remove
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  Fight re-invention!
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- scope and name spaces
  - more important but harder than in programming
  - balancing locality is tricky – definitions vs. theorems

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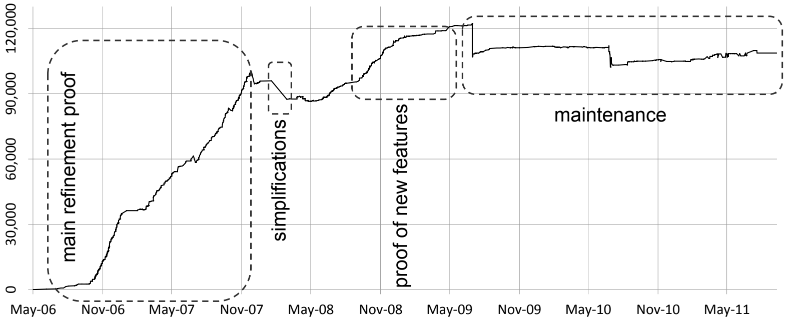
Proof Maintenance



Lines of proof over time in one L4.verified module

Gerwin's Conclusion

Proof time is short but maintenance is for life.

Proof Maintenance

Need Sophisticated Refactoring Tools

WANTED

Native **proof-refactoring tools** to

- rename constants, types, and lemmas;
- reformulate definitions or properties for more consistency;
- move lemmas for better accessibility and reusablitiy;
- disentangle dependencies;
- remove duplication.

- largely unexplored, challenging research area
- even simple renaming requires semantic analysis
- non-local changes – automation paramount

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Social and Management Aspects

From Many Brains to One Proof

- Discipline: `lemma bloody_obvious: "..."`
  - overwhelming need for meaningful names – challenging
  - self-discipline decreases over time – **tools!**
- State of Proof – regression tests
- Concurrent Development
  - compositionality under side-conditions,
    effective communication if side-conditions change
  - state the final top-level theorem first
  - continuous regression test

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Summary

Scale changes everything

Rafal:	**Proof introspection** is essential.
Matthias:	Productivity hinges on a **short edit-check cycle**.
Tim:	**Customisable automation** is crucial.
Gerwin:	**Maintenance** is for life.

Thank you
for your attention!